Sequential decision making: modeling how we interact with the world

Alan Malek

April 9th, 2016
- Omer Atun: CEO of AgilOne Predictive Marketing
- Brienne Ghafouriar: CEO of Entefy
- Jeffrey Rothschild: founder Veritas Software and Mpath Interactive, Facebook VP of Infrastructure Software
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- Alan Malek: Graduate student, Harker alumnus ’05
Goals of this talk

- What is Grad school?
- Modern sequential decision making problems
- Whet your appetite with a cool problem
- Some advice: is grad school for you?
What is a Grad school?

- A third the pay for a third the responsibilities
- Take classes for a few years
- Be a TA
- Do research
- In theory: think about how to formalize problems, prove theorems
- Or more applied: engineer solutions
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  - In theory: think about how to formalize problems, prove theorems
  - Or more applied: engineer solutions
Imagine a circle that contains all of human knowledge:

Credit: Matt Might
http://matt.might.net/articles/phd-school-in-pictures/
What is a PhD?

By the time you finish elementary school, you know a little:

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What is a PhD?

By the time you finish elementary school, you know a little:

By the time you finish high school, you know a bit more:

With a bachelor's degree, you gain a specialty:

A master's degree deepens that specialty:

Reading research papers takes you to the edge of human knowledge:

Once you're at the boundary, you focus:

You push at the boundary for a few years:

Until one day, the boundary gives way:

And, that dent you've made is called a Ph.D.:

Of course, the world looks different to you now:

So, don't forget the bigger picture:

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- Give some advice: how to prepare for grad school
Big Data

- Tons of data: \(2.5 \times 10^{18}\) bytes a day \(^1\)
- 90% of the world’s data created in the last 2 years \(^1\)
- Google: 100 billion searches a month, half from mobile \(^2\)

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\(^1\)http://www-01.ibm.com/software/data/bigdata/what-is-big-data.html
\(^2\)http://blogs.wsj.com/digits/2015/10/08/
google-says-mobile-searches-surpass-those-on-pcs/
Big Data

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- Google: 100 billion searches a month, half from mobile $^2$
- Personalization: Andrew Ng’s Coursera example

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$^2$http://blogs.wsj.com/digits/2015/10/08/
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Other side: Interactions

- Ubiquity of devices

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- Smartphone adoption rates:\(^3\)

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<thead>
<tr>
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<td>88</td>
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<td>2</td>
<td>Australia</td>
<td>77</td>
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<tr>
<td>3</td>
<td>Israel</td>
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- Sequential Interaction

Ubiquity of devices

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Sequential Interaction

Thought experiment: how did people arrange to meet before cell phones?

\textsuperscript{3} Pouchter, Jacob “Smartphone Ownership and Internet Usage Continues to Climb in Emerging Economies, Pew Research Center”. Pewglobal.org. Retrieved 2016-02-23.
Sequential Decision Making - running a newspaper

First problem: which headline to choose?
Goal: pick the headline that sells the best
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Sequential Decision Making - running a newspaper

- First problem: which headline to choose?

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Print Newspapers

1. Choose headline
Too late: by the time feedback comes, your headlines are stale.
Print Newspapers

1. Choose headline
2. Sell paper
3. See sales

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Feedback?

Too late: by the time feedback comes, your headlines are stale.
1. User arrives
Internet

1. User arrives
2. Choose article

![Diagram showing user interaction with the Internet]
1. User arrives  2. Choose article  3. Show article
1. User arrives  
2. New weights  
3. Show article  
4. Feedback
1. User arrives 2. New weights
Internet

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Internet

Sequential problem

- Unlike newspaper stand, we get repeated feedback
- Can modify our choices in *real time*

**exploration** vs. **exploitation**
Sequential problem

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**exploration** vs. **exploitation**

Continue to explore headlines until we are very sure we haven’t missed the best one.
Sequential problem

- Unlike newspaper stand, we get repeated feedback
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**exploration**

Continue to explore headlines until we are very sure we haven’t missed the best one.

**vs.**

**exploitation**

Use the headline that has been the best so far.
Sequential problem

- Unlike newspaper stand, we get repeated feedback
- Can modify our choices in \textit{real time}

**exploration** vs. **exploitation**

- Continue to explore headlines until we are very sure we haven’t missed the best one.
- Use the headline that has been the best so far.

- How can we formalize this?
Multi-Armed Bandit

Given: game length $T$, arms $1, \ldots, K$

For $t = 1, 2, \ldots, T$:
- Adversary chooses rewards $r(t, k) \in [0, 1]$
- Learner chooses an arm $k_t$
- Learner gets reward $r(t, k_t)$

Figure: game protocol
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Learner does not see rewards for other actions
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Figure: game protocol

Learner needs to randomize
Learner does not see rewards for other actions

$$\text{Regret}_T = \max_{k'} \sum_{t=1}^{T} r(t, k') - \sum_{t=1}^{T} r(t, k_t)$$

reward of best arm
Learner’s reward
Simple problem, but already interesting

Given: game length $T$, arms $1, \ldots, K$

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Figure: game protocol

- Exploitation vs. exploration
- Stochastic vs. Adversarial data
Stochastic vs. Adversarial data

- **Stochastic**
  - Most of statistics, machine learning
  - Assume that data are generated from some random variable
  - E.g. arm $k$ has mean $\mu_k$ and

$$r(t, k) = \begin{cases} 1 & \text{with probability } \mu_k \\ 0 & \text{with probability } 1 - \mu_k. \end{cases}$$

  - Past data and future data are similar
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  - Past data and future data are similar

- **Adversarial**
  - *No* assumptions on $r(t, k)$; much harder
  - Adversary could choose $r(t, k)$ based on your choices for time $1, \ldots, t - 1$ to make you do badly
  - Need regret; minimizing loss is hopeless
    $$\text{Regret}_T = \max_{k'} \sum_{t=1}^{T} r(t, k') - \sum_{t=1}^{T} r(t, k_t)$$
    - reward of best arm
    - Learner’s reward
Solutions to the bandit problem

- $\epsilon$-greedy (warm-up)
- EXP3
$\epsilon$-greedy

Given: parameter $\epsilon$, arms 1, \ldots, $K$  
For $t = 1, 2, \ldots$ :

- With probability $\epsilon$, pick arm $k_t$ uniform at random
- Otherwise, pick $k_t = \arg \max_k \sum_{s=1}^{t-1} r(s, k)$

Easy to defeat with adversarial data  
Not even optimal for stochastic data
Given: parameter $\epsilon$, arms $1, \ldots, K$

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- Easy to defeat with adversarial data
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EXP3 algorithm

Given: parameter $\eta$, arms $1, \ldots, K$
Set: $w_i(1) = 1$ for $i = 1, \ldots, K$
For $t = 1, 2, \ldots$
- Set $p_i(t) = \frac{w_i(t)}{\sum_{j=1}^{K} w_j(t)}$ for $i = 1, \ldots, K$
- Draw $k_t$ randomly proportional to $p_1(t), \ldots, p_K(t)$
- Get reward $r(t, k_t)$
- For $i = 1, \ldots, K$ set

$$\hat{r}(t, i) = \begin{cases} 
    r(t,j)/p_j(t) & \text{if } i = k_t \\
    0 & \text{otherwise,}
\end{cases}$$

$$w_j(t + 1) = w_j(t) \exp(\eta \hat{r}(t, i)) = \exp \left( \eta \sum_{s=1}^{t} \hat{r}(s, i) \right)$$
The EXP3 algorithm has the following bound:

\[
\mathbb{E} [\text{Regret}_T] = \max_{k'} \sum_{t=1}^{T} r(t, k') - \sum_{t=1}^{T} \mathbb{E} [r(t, k_t)] \\
\leq \frac{\eta TK}{2} + \frac{\log(K)}{\eta}.
\]

If we tune \( \gamma \) for \( T \),

\[
\mathbb{E} [\text{Regret}_T] \leq \sqrt{2KT \log(K)}.
\]
Theorem

The EXP3 algorithm has the following bound:

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If we tune $\gamma$ for $T$, 

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If we tune \( \gamma \) for \( T \),

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Regret per round \( \rightarrow 0 \)

lower bound of \( \Omega(\sqrt{KT}) \)
Recap

- Started with real world problem
- Abstracted into Multi-Armed Bandit framework
- Proposed algorithms
- Proved upper bounds on their regret
- Compared to lower bounds
Goals of this talk

- What is Grad school?
- Modern sequential decision making problems
- Whet your appetite with a cool problem
- Give some advice: how to prepare for grad school
Things I wish I knew in college

- Your professors want to talk to you and meet undergrads. Go to their office hours, group meetings, ask about problems.
- If research might be for you, get involved early (sophomore).
- You will need three good letters to get a good grad school.
- Sometimes you will learn more from a research project than an extra class.
- Failing is part of it!
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*Failing is part of it!*
Is a PhD for you?

Pros:
- Research is also rewarding; occasionally fun
- Very independent
- Become an expert in something!
- Exciting problems
Is a PhD for you?

Pros:
- Research is also rewarding; occasionally fun
- Very independent
- Become an expert in something!
- Exciting problems

Cons:
- Research is frustrating; many more failed attempts
- Long hours, little pay
- Few jobs require a PhD
- Don’t escape politics
Thank you!